3) (Marks: 30)

a. Find first three terms of Taylor series of $f(x) = \sec x$ at $c = \frac{\pi}{3}$.

b. Given points are $P(4,0,0)$, $Q(0,5,0)$, $R(0,0,2)$. Find the area of the triangle PQR.

c. Show that the limit $\lim_{(x,y)\to(0,0)} \frac{4x^3y}{2x^4+3y^4}$ does not exist.

d. Find the directional derivative of $f(x,y) = \tan^{-1}\left(\frac{y}{x}\right)$ at the point $(4,-4)$ in the direction of the vector $\vec{a} =< 2,-3 >$.

e. A constant force of magnitude 22 lb has the same direction as the vector $\vec{a} =< 4,7,4 >$. If the distance is measured in feet, find the work done if the point of application moves along a straight line from $(1,1,1)$ to $(3,5,4)$.

4) (Marks: 40)

a. Sketch the graph of the polar equation $r = 4(1-\sin\theta)$.

b. Determine whether given lines intersect or not. If they intersect then find their point of intersection.

$$x = 3+t, \quad y = 2-4t, \quad z = t$$

$$x = 4-s, \quad y = 3+s, \quad z = -2+3s$$

c. Sketch the level curves of $f(x,y) = x^2 - y$ when $k = -2, 0, 3$

d. If $w = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$. Show that

$$(w_x)^2 + (w_y)^2 = (w_r)^2 + \frac{1}{r^2}(w_\theta)^2.$$

e. Find Extrema and saddle points of the function

$$f(x,y) = \frac{1}{2}x^4 - 2x^3 + 4xy + y^2.$$

P.T.O.

COMSATS University Islamabad, Abbottabad Campus
Department of Mathematics
Final-term Exam, Spring 2025

Class: BSE 4(B, D)
Subject: Multivariable Calculus
Total Time Allowed: 3 hours
Name [redacted]

Date: 16/07/2025
Teacher: Dr. Usman Ashraf
Max Marks: 100
Reg. # [redacted]

1) (Marks: 10)

a. Given $\vec{a} =< -2, 3, 1 >$ and $\vec{b} =< 7, 4, 5 >$. Find $\vec{a} \bullet \vec{b}$.

b. Find $f(0,1)$, where $f(x,y) = \frac{xy}{x-2y}$.

c. Evaluate the limit $\lim_{(x,y)\to(2,0)} (x^3 - 4xy^2 + 5y - 7)$.

d. Find f_{xy} where $f(x,y) = x^3y^2 - 2x^2y + 3x$.

e. Find ∇f (gradient of f), where $f(x,y) = 2 + x^2 + \frac{1}{4}y^2$.

2) (Marks: 20)

a. Evaluate the integral $\int \tan^{-1} x \, dx$.

b. Convert the polar equation $r^2 \cos 2\theta = 1$ into an equation in rectangular coordinates.

c. Find a vector that same direction and one-half the magnitude as of the vector $< -6, 3 >$.

d. Find the center and radius of the sphere

$$x^2 + y^2 + z^2 - 8x + 8y + 16 = 0.$$

e. A flat metal plate is lies on an xy-plane such that the temperature T at any point (x,y) is given by $T(x,y) = 10(x^2 + y^2)^2$. Find instantaneous rate of change of T with respect to the distance at the point $(1, 2)$ in the direction of y-axis.

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