COMSATS University Islamabad, Abbottabad Campus
Department of Mathematics
Final Examination Fall 2024

Class: BCS 4(A/B/C)
Subject: Multivariable Calculus
Time Allowed: 180 Minutes
Name: _____

Date: January 21, 2025
Instructor: Dr. Usman Ashraf
Max Marks: 100
Registration: _____

Q. No. 1. Solve the following questions. **(16 marks)**

1A. If $f(x,y,z) = yz^3 - 2x^2$ then find ∇f.

1B. If $f(x,y,z) = xy^3 - 2x^2 + \pi$ then find f_z.

1C. If $f(x,y) = 2x - y^2$ then find $f(-2,5)$ and $f(0,-2)$.

1D. Position vector of a moving point is given by $\mathbf{r}(t) = e^{2t}\mathbf{i} + e^{-t}\mathbf{j}$, find its velocity at time $t = 1$.

1E. Suppose $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{j} + \mathbf{k}$, then find $\mathbf{a}\cdot\mathbf{b}$.

1F. Suppose $\mathbf{a} = \mathbf{i} + \mathbf{j}$ and $\mathbf{b} = \mathbf{j} + \mathbf{k}$, then find $\mathbf{a}+\mathbf{b}$.

1G. Find the radius of the sphere $4x^2 + 4y^2 + 4z^2 = 1$.

1H. Evaluate the limit $\lim_{n\to\infty}\{1 + (0.1)^n\}$.

Q. No. 2. Solve the following questions. **(24 marks)**

2A. Find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{\partial y}$ when $w = u\sin v$, $u = x^2 + y^2$, $v = xy$.

2B. A flat metal plate lies on an xy-plane such that the temperature T at any point (x,y) is given by $T = 10(x^2 + y^2)^2$, where T is in degrees and x and y are in centimeters. Find the instantaneous rate of change of T with respect to distance at $(1,2)$ in the direction of x-axis and y-axis.

2C. Evaluate the limit $\lim_{(x,y)\to(0,0)} \frac{3x^3 - 2x^2y + 3y^2x - 2y^3}{x^2 + y^2}$.

2D. Find equations of the line through the points $P(5,-2,4)$ and $Q(2,6,1)$.

2E. Find an equation of the plane through the point $P(2,5,-6)$ and parallel to the plane $3x - y + 2z = 10$.

2F. Determine the convergence/divergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+9}$.

Q. No. 3. Solve the following questions. **(36 marks)**

3A. Find the directional derivative of the function $f(x,y) = x^2 - 5xy + 3y^2$ at the point $P(3,-1)$ in the direction of unit vector $\mathbf{u} = \left(\frac{\sqrt{2}}{2}\right)(\mathbf{i} + \mathbf{j})$.

3B. If $w = f(x,y)$, where $x = r\cos\theta$ and $y = r\sin\theta$, show that

$$\left(\frac{\partial w}{\partial x}\right)^2 + \left(\frac{\partial w}{\partial y}\right)^2 = \left(\frac{\partial w}{\partial r}\right)^2 + \frac{1}{r^2}\left(\frac{\partial w}{\partial \theta}\right)^2.$$

3C. A small cart weighing 100 lb is pushed up an incline that makes an angle of 30° with the horizontal. Find the work done against the gravity in pushing the cart a distance of 80 ft.

3D. Suppose $\mathbf{a} = 14\mathbf{i} - 15\mathbf{j} + 6\mathbf{k}$. Find a vector that has opposite direction as $\mathbf{a}$, and one-third of magnitude of $\mathbf{a}$. Also find a vector of magnitude 2 that has the same direction as $\mathbf{a}$.

3E. Find scalars p and q such that $p\langle 3,-1\rangle + q\langle 4,3\rangle = \langle -6,\cdots 11\rangle$.

3F. A parametrized curve is given by $x = t^2 + 1$, $y = t^2 - 1$, $-2 \le t \le 2$. Sketch the given curve and find an equation in x and y that has the same graph as of given curve.

Q. No. 4. Solve the following questions. **(24 marks)**

4A. Find the extrema and saddle points of the function $f(x,y) = x^4 + y^3 + 32x - 9y$.

4B. Find the area of the triangle PQR where $P(4,0,0)$, $Q(0,5,0)$ and $R(0,0,2)$.

4C. Evaluate the integral $\int \frac{1}{x\sqrt{9+x^2}}\,dx$.

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